

Rough Intuitionistic Fuzzy Ideals in a Ring

G. Senthil Kumar, V. Selvan

Abstract— In this paper basic notion of rough intuitionistic fuzzy set in rings are given and some of its basic properties are discussed. We introduce the notion of rough intuitionistic fuzzy ideal (prime ideal) in a ring and we discuss the relationship between upper (lower) rough prime ideal and upper (lower) approximations of their homomorphic images.

Index Terms— intuitionistic fuzzy set, rough set, intuitionistic fuzzy ideal, rough intuitionistic fuzzy prime ideal.



1 INTRODUCTION

The theory of fuzzy sets was introduced by Zadeh [22] in 1965 as an attempt to study the vagueness and uncertainty in real world problems. A. Rosenfeld [16] applied the notion of fuzzy sets and introduced the notion of fuzzy subgroups in groups. Since then various classical algebraic systems have been fuzzified.

The theory of rough sets introduced by Z. Pawlak [14] in 1982 is another independent method to deal the vagueness and uncertainty. Z. Pawlak used equivalence classes in a set as the building blocks for the construction of lower and upper approximations of a set.

R. Biswas and S. Nanda [5] introduced the concept of lower and upper approximation of a subgroup of a group. N. Kuroki [12] gave the notion of rough ideal in a semigroup. The concept of rough group was also considered by N. Kuroki and J. N. Moderson [13]. B. Davvaz [6] has introduced roughness in rings. O. Kazanci and B. Davvaz [10] introduced the notion of rough prime ideals and rough primary ideals in rings. The authors V. Selvan and G. Senthil Kumar [17, 18] have studied the lower and upper approximations of ideals and fuzzy ideals in a semiring.

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy set was introduced by K. T. Atanassov in [1]. R. Biswas [4] used the concept of intuitionistic fuzzy set to the theory of groups and studied the intuitionistic fuzzy subgroups of a group. The concept of intuitionistic fuzzy R-subgroup of a near ring was given by Y. H. Yon, Y. B. Jun and K. H. Kim [21]. The concept of (α, β) -cut of intuitionistic fuzzy ideals in a ring was given by D. K. Basnet [3]. The notion of rough intuitionistic fuzzy ideal in a semi group was given by J. Gosh and T. K. Samanta [9].

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In this paper, in section 3 we prove that any intuitionistic fuzzy subring (ideal) of a ring is an upper (lower) approximation intuitionistic fuzzy subring (ideal) of the ring.

In section 4 of this paper, we introduce the rough intuitionistic fuzzy prime ideal of a ring. Also we discuss the relationship between upper and lower intuitionistic fuzzy ideals (prime ideals) and the upper and lower approximation of their homomorphic images.

2 PRELIMINARIES

Definition 2.1 [14] Let (U, θ) be an approximation space, where U is the non-empty universe, θ is an equivalence relation and let X be any non-empty subset of U . Then the sets

$$\theta_*(X) = \{x \in U \mid [x]_\theta \subseteq X\} \text{ and}$$

$$\theta^*(X) = \{x \in U \mid [x]_\theta \cap X \neq \emptyset\}$$

are respectively called the lower approximation and upper approximation of the set X with respect to θ , where $[x]_\theta$ denotes the equivalence class containing the element $x \in X$ with respect to θ . X is called θ -definable if $\theta_*(X) = \theta^*(X)$. If $\theta_*(X) \neq \theta^*(X)$ then X is called rough set with respect to θ .

Definition 2.2 [1] An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A = \{ \langle x, \mu_A(x), \lambda_A(x) \rangle \mid x \in X \}$, where the functions $\mu_A : X \rightarrow [0, 1]$ and $\lambda_A : X \rightarrow [0, 1]$ denote the degree of membership and degree of non-membership of the element $x \in X$ to A , respectively and satisfy $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. The family of all intuitionistic fuzzy set in X is denoted by $\text{IFS}(X)$.

An intuitionistic fuzzy set $A = \{ \langle x, \mu_A(x), \lambda_A(x) \rangle \mid x \in X \}$ in X can be identified to an ordered pair (μ_A, λ_A) in $I^X \times I^X$. For the sake of simplicity, we

shall use the symbol $A = (\mu_A, \lambda_A)$ for the IFS $A = \{ \langle x, \mu_A(x), \lambda_A(x) \rangle \mid x \in X \}$.

Definition 2.3 [1] If $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are any two IFS in X , then

- (i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$, for all $x \in X$.
- (ii) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\lambda_A(x) = \lambda_B(x)$, for all $x \in X$.
- (iii) $A^c = (\lambda_A, \mu_A)$
- (iv) $A \cap B = (\mu_A \cap \mu_B, \lambda_A \cup \lambda_B)$, where $(\mu_A \cap \mu_B)(x) = \mu_A(x) \wedge \mu_B(x)$ and $(\lambda_A \cup \lambda_B)(x) = \lambda_A(x) \vee \lambda_B(x)$, for all $x \in X$.
- (v) $A \cup B = (\mu_A \cup \mu_B, \lambda_A \cap \lambda_B)$, where $(\mu_A \cup \mu_B)(x) = \mu_A(x) \vee \mu_B(x)$ and $(\lambda_A \cap \lambda_B)(x) = \lambda_A(x) \wedge \lambda_B(x)$, for all $x \in X$.

Throughout this paper R denotes a ring.

Definition 2.4 [2] Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are any two IFS of R . Then their product $A.B$ is defined by

$$A.B = (\mu_A \cdot \mu_B, \lambda_A \cdot \lambda_B),$$

$$\text{where } (\mu_A \cdot \mu_B)(x) = \bigvee_{x=yz} [\mu_A(y) \wedge \mu_B(z)]$$

$$\text{and } (\lambda_A \cdot \lambda_B)(x) = \bigwedge_{x=yz} [\lambda_A(y) \vee \lambda_B(z)], \text{ for all } x \in R.$$

Definition 2.5 [2] Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are any two IFS of R . Then their sum $A+B$ is defined by

$$A+B = (\mu_A + \mu_B, \lambda_A + \lambda_B)$$

$$\text{where } (\mu_A + \mu_B)(x) = \bigvee_{x=y+z} [\mu_A(y) \wedge \mu_B(z)]$$

$$\text{and } (\lambda_A + \lambda_B)(x) = \bigwedge_{x=y+z} [\lambda_A(y) \vee \lambda_B(z)], \text{ for all } x \in R.$$

Definition 2.6 [2] An IFS $A = (\mu_A, \lambda_A)$ of a ring R is said to be an intuitionistic fuzzy ideal of R if for all $x, y \in R$,

- (i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$
- (ii) $\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$
- (iii) $\lambda_A(x-y) \leq \lambda_A(x) \vee \lambda_A(y)$
- (iv) $\lambda_A(xy) \leq \lambda_A(x) \wedge \lambda_A(y)$

Definition 2.7 [2] An IFS $A = (\mu_A, \lambda_A)$ of a ring R is said to be an intuitionistic fuzzy subring of R if for all $x, y \in R$,

- (i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$

- (ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$
- (iii) $\lambda_A(x-y) \leq \lambda_A(x) \vee \lambda_A(y)$
- (iv) $\lambda_A(xy) \leq \lambda_A(x) \vee \lambda_A(y)$

Definition 2.8 Let R and R' be two rings and $f: R \rightarrow R'$ be a homomorphism, and $A = (\mu_A, \lambda_A)$ and $A' = (\mu_{A'}, \lambda_{A'})$ be IFS of R and R' respectively, then the image $f(A)$ and the inverse image $f^{-1}(A')$ are defined as follows:

$$f(A) = (f(\mu_A), f(\lambda_A)), \text{ where}$$

$$f(\mu_A)(y) = \sup \{ \mu_A(x) : x \in f^{-1}(y) \} \quad \text{if } f^{-1}(y) \neq \emptyset$$

$$= 0 \text{ if } f^{-1}(y) = \emptyset$$

$$f(\lambda_A)(y) = \inf \{ \lambda_A(x) : x \in f^{-1}(y) \} \quad \text{if } f^{-1}(y) \neq \emptyset$$

$$= 0 \text{ if } f^{-1}(y) = \emptyset, \quad \forall y \in R'$$

$$\text{and } f^{-1}(A') = (f^{-1}(\mu_{A'}), f^{-1}(\lambda_{A'}))$$

$$\text{where } f^{-1}(\mu_{A'})(x) = \mu_{A'}(f(x)),$$

$$f^{-1}(\lambda_{A'})(x) = \lambda_{A'}(f(x)), \quad \forall x \in R.$$

3 ROUGH INTUITIONISTIC FUZZY SUBSETS (RIFS) IN A RING

Definition 3.1 [10] Let θ be an equivalence relation on R , then θ is called a full congruence relation if $(a, b) \in \theta$ implies

$$(a+x, b+x), (ax, bx) \text{ and } (xa, xb) \in \theta, \text{ for all } x \in R.$$

We denote the θ -congruence class containing the element $x \in R$ by $[x]_\theta$.

Theorem 3.2 [10] Let θ be a full congruence relation on a ring R . If $a, b \in R$ then

$$(i) \quad [a]_\theta + [b]_\theta = [a+b]_\theta$$

$$(ii) \quad [-a]_\theta = -[a]_\theta$$

$$(iii) \quad \{xy \mid x \in [a]_\theta, y \in [b]_\theta\} \subseteq [ab]_\theta$$

A full congruence relation θ on R is called complete if $[ab]_\theta = [a]_\theta [b]_\theta$, for all $a, b \in R$.

Definition 3.3 Let θ be a full congruence relation on R and $A = (\mu_A, \lambda_A)$ be a IFS of R . Then the IFS $\theta_*(A) = (\theta_*(\mu_A), \theta_*(\lambda_A))$ and $\theta^*(A) = (\theta^*(\mu_A), \theta^*(\lambda_A))$ are respectively called θ -lower and θ -upper approximations of the IFS A ,

$$\text{where } \theta_*(\mu_A)(x) = \bigwedge_{a \in [x]_\theta} \mu_A(a), \quad \theta_*(\lambda_A)(x) = \bigvee_{a \in [x]_\theta} \lambda_A(a),$$

$\theta^*(\mu_A)(x) = \bigvee_{a \in [x]_\theta} \mu_A(a)$, $\theta^*(\lambda_A)(x) = \bigwedge_{a \in [x]_\theta} \lambda_A(a)$, for all $x \in R$. that, for every $x \in R$,
 $(\theta^*(\mu_A) + \theta^*(\mu_B))(x) \leq \theta^*(\mu_A + \mu_B)(x)$ and
 $(\theta^*(\lambda_A) + \theta^*(\lambda_B))(x) \geq \theta^*(\lambda_A + \lambda_B)(x)$

For an IFS $A = (\mu_A, \lambda_A)$ of R , $\theta(A) = (\theta_*(A), \theta^*(A))$ is called rough intuitionistic fuzzy set with respect to θ if $\theta_*(A) \neq \theta^*(A)$.

Theorem 3.4 Let θ, ϕ be two full congruence relations on R . If $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are any two IFS of R , then the following hold:

- (i) $\theta_*(A) \subseteq A \subseteq \theta^*(A)$
- (ii) $\theta_*(\theta_*(A)) = \theta_*(A)$
- (iii) $\theta^*(\theta^*(A)) = \theta^*(A)$
- (iv) $\theta^*(\theta_*(A)) = \theta_*(A)$
- (v) $\theta_*(\theta^*(A)) = \theta^*(A)$
- (vi) $(\theta^*(A^c))^c = \theta_*(A)$
- (vii) $(\theta_*(A^c))^c = \theta^*(A)$
- (viii) $\theta_*(A \cap B) = \theta_*(A) \cap \theta_*(B)$
- (ix) $\theta^*(A \cap B) \subseteq \theta^*(A) \cap \theta^*(B)$
- (x) $\theta^*(A \cup B) = \theta^*(A) \cup \theta^*(B)$
- (xi) $\theta_*(A \cup B) \supseteq \theta_*(A) \cup \theta_*(B)$
- (xii) $A \subseteq B \Rightarrow \theta_*(A) \subseteq \theta_*(B)$
- (xiii) $A \subseteq B \Rightarrow \theta^*(A) \subseteq \theta^*(B)$
- (xiv) $\theta \subseteq \phi \Rightarrow \theta_*(A) \supseteq \phi_*(A)$
- (xv) $\theta \subseteq \phi \Rightarrow \theta^*(A) \subseteq \phi^*(A)$.

Proof. It is straight forward.

Theorem 3.5 Let θ be a congruence relation on R . If A, B are any two IFS of R , then $\theta^*(A) + \theta^*(B) \subseteq \theta^*(A + B)$.

Proof. Since θ is a congruence relation on R , $[a]_\theta + [b]_\theta \subseteq [a + b]_\theta$, $\forall a, b \in R$.

Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be any two IFS of R . Then $\theta^*(A) + \theta^*(B) = (\theta^*(\mu_A) + \theta^*(\mu_B), \theta^*(\lambda_A) + \theta^*(\lambda_B))$

and $\theta^*(A + B) = (\theta^*(\mu_A + \mu_B), \theta^*(\lambda_A + \lambda_B))$

To prove $\theta^*(A) + \theta^*(B) \subseteq \theta^*(A + B)$, it is enough to prove

$$\begin{aligned} (\theta^*(\mu_A) + \theta^*(\mu_B))(x) &= \bigvee_{x=y+z} [\theta^*(\mu_A)(y) \wedge \theta^*(\mu_B)(z)] \\ &= \bigvee_{x=y+z} \left[\left(\bigvee_{a \in [y]_\theta} \mu_A(a) \right) \wedge \left(\bigvee_{b \in [z]_\theta} \mu_B(b) \right) \right] \\ &= \bigvee_{x=y+z} \left[\bigvee_{\substack{a \in [y]_\theta, \\ b \in [z]_\theta}} (\mu_A(a) \wedge \mu_B(b)) \right] \\ &\leq \bigvee_{x=y+z} \left[\bigvee_{a+b \in [y+z]_\theta} (\mu_A(a) \wedge \mu_B(b)) \right] \\ &= \bigvee_{a+b \in [x]_\theta} [\mu_A(a) \wedge \mu_B(b)] \\ &\quad , \text{ since } y+z=x \\ &= \bigvee_{\substack{\alpha \in [x]_\theta \\ \alpha=a+b}} [\mu_A(a) \wedge \mu_B(b)] \\ &= \bigvee_{\alpha \in [x]_\theta} \bigvee_{\alpha=a+b} [\mu_A(a) \wedge \mu_B(b)] \\ &= \bigvee_{\alpha \in [x]_\theta} [\mu_A + \mu_B](\alpha) \\ &= \theta^*(\mu_A + \mu_B)(x) \\ (\theta^*(\lambda_A) + \theta^*(\lambda_B))(x) &= \bigwedge_{x=y+z} [\theta^*(\lambda_A)(y) \vee \theta^*(\lambda_B)(z)] \\ &= \bigwedge_{x=y+z} \left[\left(\bigwedge_{a \in [y]_\theta} \lambda_A(a) \right) \vee \left(\bigwedge_{b \in [z]_\theta} \lambda_B(b) \right) \right] \\ &= \bigwedge_{x=y+z} \left[\bigwedge_{\substack{a \in [y]_\theta, \\ b \in [z]_\theta}} (\lambda_A(a) \vee \lambda_B(b)) \right] \\ &\geq \bigwedge_{x=y+z} \left[\bigwedge_{a+b \in [y+z]_\theta} (\lambda_A(a) \vee \lambda_B(b)) \right] \\ &\quad , \text{ since } a+b \in [y]_\theta + [z]_\theta \subseteq [y+z]_\theta \\ &= \bigwedge_{a+b \in [x]_\theta} [\lambda_A(a) \vee \lambda_B(b)] \\ &= \bigwedge_{\alpha \in [x]_\theta} \bigwedge_{\alpha=a+b} [\lambda_A(a) \vee \lambda_B(b)] \\ &= \bigwedge_{\alpha \in [x]_\theta} [\lambda_A + \lambda_B](\alpha) \\ &= \theta^*(\lambda_A + \lambda_B)(x) \end{aligned}$$

Thus we have, $\theta^*(A) + \theta^*(B) \subseteq \theta^*(A + B)$. Equality holds, if θ is a full congruence relation.

Theorem 3.6 Let θ be a complete congruence relation on R . If A, B are any two IFS of R , then $\theta_*(A) + \theta_*(B) \subseteq \theta_*(A + B)$.

Proof. Since θ is complete congruence relation on R ,

$$[a]_{\theta} + [b]_{\theta} = [a+b]_{\theta}, \quad \forall a, b \in R.$$

Let $A = (\mu_A, \lambda_A)$, $B = (\mu_B, \lambda_B)$ be any two IFS of R . Then

$$\theta_*(A) + \theta_*(B) = (\theta_*(\mu_A) + \theta_*(\mu_B), \theta_*(\lambda_A) + \theta_*(\lambda_B)) \quad \text{and}$$

$$\theta_*(A+B) = (\theta_*(\mu_A + \mu_B), \theta_*(\lambda_A + \lambda_B))$$

To show $\theta_*(A) + \theta_*(B) \subseteq \theta_*(A+B)$,

we have to prove that, for every $x \in R$,

$$(\theta_*(\mu_A) + \theta_*(\mu_B))(x) \leq \theta_*(\mu_A + \mu_B)(x) \quad \text{and}$$

$$(\theta_*(\lambda_A) + \theta_*(\lambda_B))(x) \geq \theta_*(\lambda_A + \lambda_B)(x).$$

Now,

$$\begin{aligned} (\theta_*(\mu_A) + \theta_*(\mu_B))(x) &= \bigvee_{x=y+z} [\theta_*(\mu_A)(y) \wedge \theta_*(\mu_B)(z)] \\ &= \bigvee_{x=y+z} \left[\left(\bigwedge_{a \in [y]_{\theta}} \mu_A(a) \right) \wedge \left(\bigwedge_{b \in [z]_{\theta}} \mu_B(b) \right) \right] \\ &= \bigvee_{x=y+z} \left[\bigwedge_{\substack{a \in [y]_{\theta}, \\ b \in [z]_{\theta}}} (\mu_A(a) \wedge \mu_B(b)) \right] \\ &\leq \bigvee_{x=y+z} \left[\bigwedge_{\substack{a \in [y]_{\theta}, \\ b \in [z]_{\theta}}} \bigvee_{a+b=\alpha+\beta} (\mu_A(\alpha) \wedge \mu_B(\beta)) \right] \\ &= \bigvee_{x=y+z} \left[\bigwedge_{\substack{a \in [y]_{\theta}, \\ b \in [z]_{\theta}}} (\mu_A + \mu_B)(a+b) \right] \\ &= \bigvee_{x=y+z} \left[\bigwedge_{a+b \in [y+z]_{\theta}} (\mu_A + \mu_B)(a+b) \right] \\ &= \bigvee_{x=y+z} \theta_*(\mu_A + \mu_B)(y+z) \\ &= \theta_*(\mu_A + \mu_B)(x) \\ (\theta_*(\lambda_A) + \theta_*(\lambda_B))(x) &= \bigwedge_{x=y+z} [\theta_*(\lambda_A)(y) \vee \theta_*(\lambda_B)(z)] \\ &= \bigwedge_{x=y+z} \left[\left(\bigvee_{a \in [y]_{\theta}} \lambda_A(a) \right) \vee \left(\bigvee_{b \in [z]_{\theta}} \lambda_B(b) \right) \right] \\ &= \bigwedge_{x=y+z} \left[\bigvee_{\substack{a \in [y]_{\theta}, \\ b \in [z]_{\theta}}} (\lambda_A(a) \vee \lambda_B(b)) \right] \\ &\geq \bigwedge_{x=y+z} \left[\bigvee_{\substack{a \in [y]_{\theta}, \\ b \in [z]_{\theta}}} \left(\bigwedge_{a+b=\alpha+\beta} (\lambda_A(a) \vee \lambda_B(b)) \right) \right] \end{aligned}$$

$$\begin{aligned} &= \bigwedge_{x=y+z} \left[\bigvee_{\substack{a \in [y]_{\theta}, \\ b \in [z]_{\theta}}} (\lambda_A + \lambda_B)(a+b) \right] \\ &= \bigwedge_{x=y+z} \bigvee_{a+b \in [y+z]_{\theta}} (\lambda_A + \lambda_B)(a+b) \\ &= \bigwedge_{x=y+z} \theta_*(\lambda_A + \lambda_B)(y+z) \\ &= \theta_*(\lambda_A + \lambda_B)(x) \end{aligned}$$

Thus we have $\theta_*(A) + \theta_*(B) \subseteq \theta_*(A+B)$.

Theorem 3.7 Let θ be a full congruence relation on R . If A, B are any two IFS of R , then $\theta^*(A), \theta^*(B) \subseteq \theta^*(A.B)$.

Proof.

Since θ is a full congruence relation on R ,

$$\{xy \mid x \in [a]_{\theta}, y \in [b]_{\theta}\} \subseteq [ab]_{\theta}, \quad \forall a, b \in R.$$

Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be any two IFS of R .

Then $\theta^*(A), \theta^*(B) = (\theta^*(\mu_A), \theta^*(\mu_B), \theta^*(\lambda_A), \theta^*(\lambda_B))$ and

$$\theta^*(A.B) = (\theta^*(\mu_A \cdot \mu_B), \theta^*(\lambda_A \cdot \lambda_B))$$

To show $\theta^*(A), \theta^*(B) \subseteq \theta^*(A.B)$, we have to prove that for all $x \in R$,

$$(\theta^*(\mu_A), \theta^*(\mu_B))(x) \leq \theta^*(\mu_A \cdot \mu_B)(x) \quad \text{and}$$

$$(\theta^*(\lambda_A), \theta^*(\lambda_B))(x) \geq \theta^*(\lambda_A \cdot \lambda_B)(x).$$

Now for all $x \in R$,

$$\begin{aligned} (\theta^*(\mu_A), \theta^*(\mu_B))(x) &= \bigvee_{x=yz} [\theta^*(\mu_A)(y) \wedge \theta^*(\mu_B)(z)] \\ &= \bigvee_{x=yz} \left[\left(\bigvee_{a \in [y]_{\theta}} \mu_A(a) \right) \wedge \left(\bigvee_{b \in [z]_{\theta}} \mu_B(b) \right) \right] \\ &= \bigvee_{x=yz} \left[\bigvee_{\substack{a \in [y]_{\theta}, \\ b \in [z]_{\theta}}} (\mu_A(a) \wedge \mu_B(b)) \right] \\ &\leq \bigvee_{x=yz} \left[\bigvee_{ab \in [yz]_{\theta}} (\mu_A(a) \wedge \mu_B(b)) \right] \\ &= \bigvee_{ab \in [x]_{\theta}} (\mu_A(a) \wedge \mu_B(b)) \\ &= \bigvee_{\substack{\alpha \in [x]_{\theta}, \\ \alpha=ab}} (\mu_A(a) \wedge \mu_B(b)) \\ &= \bigvee_{\substack{\alpha \in [x]_{\theta}, \\ \alpha=ab}} \bigvee_{\alpha=ab} (\mu_A(a) \wedge \mu_B(b)) \\ &= \bigvee_{\alpha \in [x]_{\theta}} (\mu_A \cdot \mu_B)(\alpha) \\ &= \theta^*(\mu_A \cdot \mu_B)(x) \end{aligned}$$

Again,

$$\begin{aligned}
 (\theta^*(\lambda_A) \cdot \theta^*(\lambda_B))(x) &= \bigwedge_{x=yz} [\theta^*(\lambda_A)(y) \vee \theta^*(\lambda_B)(z)] \\
 &= \bigwedge_{x=yz} \left[\left(\bigwedge_{a \in [y]_\theta} \lambda_A(a) \right) \vee \left(\bigwedge_{b \in [z]_\theta} \lambda_B(b) \right) \right] \\
 &= \bigwedge_{x=yz} \left[\bigwedge_{\substack{a \in [y]_\theta, \\ b \in [z]_\theta}} (\lambda_A(a) \vee \lambda_B(b)) \right] \\
 &\geq \bigwedge_{x=yz} \left[\bigwedge_{ab \in [yz]_\theta} (\lambda_A(a) \vee \lambda_B(b)) \right] \\
 &= \bigwedge_{ab \in [x]_\theta} (\lambda_A(a) \vee \lambda_B(b)) \\
 &= \bigwedge_{\alpha \in [x]_\theta, \alpha=ab} (\lambda_A(a) \vee \lambda_B(b)) \\
 &= \bigwedge_{\alpha \in [x]_\theta} \bigwedge_{\alpha=ab} (\lambda_A(a) \vee \lambda_B(b)) \\
 &= \bigwedge_{\alpha \in [x]_\theta} (\lambda_A \cdot \lambda_B)(\alpha) \\
 &= \theta^*(\lambda_A \cdot \lambda_B)(x)
 \end{aligned}$$

Thus we have, $\theta^*(A) \cdot \theta^*(B) \subseteq \theta^*(A \cdot B)$.

Theorem 3.8 Let θ be a complete congruence relation on R . If A and B are any two IFS of R , then

$$\theta_*(A) \cdot \theta_*(B) \subseteq \theta_*(A \cdot B).$$

Proof. The proof is similar to Theorem 3.6.

Theorem 3.9 Let θ, ϕ be two congruence relations on R . If A is an IFS on R , then $(\theta \cap \phi)^*(A) \subseteq \theta^*(A) \cap \phi^*(A)$.

Proof.

Clearly, $\theta \cap \phi$ is a congruence relation on R and $\theta \cap \phi \subseteq \theta$, $\theta \cap \phi \subseteq \phi$.

Let $A = (\mu_A, \lambda_A)$ be an IFS of R .

Then by Theorem 3.4 (xv), we obtain

$$(\theta \cap \phi)^*(A) \subseteq \theta^*(A) \text{ and } (\theta \cap \phi)^*(A) \subseteq \phi^*(A).$$

Therefore, $(\theta \cap \phi)^*(A) \subseteq \theta^*(A) \cap \phi^*(A)$.

Theorem 3.10 Let θ, ϕ be two congruence relations on R . If A is an IFS on R , then $(\theta \cap \phi)_*(A) \supseteq \theta_*(A) \cup \phi_*(A)$.

Proof.

Clearly, $\theta \cap \phi$ is a congruence relation on R and $\theta \cap \phi \subseteq \theta$, $\theta \cap \phi \subseteq \phi$.

Let $A = (\mu_A, \lambda_A)$ be an IFS of R .

Then by Theorem 3.4 (xiv), we obtain

$$(\theta \cap \phi)_*(A) \supseteq \theta_*(A) \text{ and } (\theta \cap \phi)_*(A) \supseteq \phi_*(A).$$

Therefore, $(\theta \cap \phi)_*(A) \supseteq \theta_*(A) \cup \phi_*(A)$.

Theorem 3.11 Let θ be a full congruence relation on R . Then

(a) If A is an intuitionistic fuzzy subring of R , then A is an upper rough intuitionistic fuzzy subring of R .

(b) If A is an intuitionistic fuzzy ideal of R , then A is an upper rough intuitionistic fuzzy ideal of R .

Proof.

(a) Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy subring of R .

$$\text{Then } \theta^*(A) = (\theta^*(\mu_A), \theta^*(\lambda_A)).$$

Now for all $x, y \in R$.

$$(i) \quad \theta^*(\mu_A)(x - y) = \bigvee_{z \in [x-y]_\theta} \mu_A(z)$$

$$= \bigvee_{z \in [x]_\theta - [y]_\theta} \mu_A(z)$$

Since θ is full congruence relation on R ,

$$= \bigvee_{a-b \in [x]_\theta - [y]_\theta} \mu_A(a-b)$$

$$\geq \bigvee_{\substack{a \in [x]_\theta, \\ b \in [y]_\theta}} [\mu_A(a) \wedge \mu_A(b)]$$

$$= \left(\bigvee_{a \in [x]_\theta} \mu_A(a) \right) \wedge \left(\bigvee_{b \in [y]_\theta} \mu_A(b) \right)$$

$$= \theta^*(\mu_A)(x) \wedge \theta^*(\mu_A)(y)$$

$$\theta^*(\lambda_A)(x - y) = \bigwedge_{z \in [x-y]_\theta} \lambda_A(z)$$

$$= \bigwedge_{z \in [x]_\theta - [y]_\theta} \lambda_A(z)$$

$$= \bigwedge_{a-b \in [x]_\theta - [y]_\theta} \lambda_A(a-b)$$

$$\leq \bigwedge_{a \in [x]_\theta, b \in [y]_\theta} [\lambda_A(a) \vee \lambda_A(b)]$$

$$= \left(\bigwedge_{a \in [x]_\theta} \lambda_A(a) \right) \vee \left(\bigwedge_{b \in [y]_\theta} \lambda_A(b) \right)$$

$$= \theta^*(\lambda_A)(x) \vee \theta^*(\lambda_A)(y)$$

$$(ii) \quad \theta^*(\mu_A)(xy) = \bigvee_{z \in [xy]_\theta} \mu_A(z)$$

$$\geq \bigvee_{z \in [x]_\theta [y]_\theta} \mu_A(z)$$

$$= \bigvee_{ab \in [x]_\theta [y]_\theta} \mu_A(ab)$$

$$\geq \bigvee_{a \in [x]_\theta, b \in [y]_\theta} [\mu_A(a) \wedge \mu_A(b)]$$

$$= \left(\bigvee_{a \in [x]_\theta} \mu_A(a) \right) \wedge \left(\bigvee_{b \in [y]_\theta} \mu_A(b) \right)$$

$$= \theta^*(\mu_A)(x) \wedge \theta^*(\mu_A)(y)$$

and

$$\begin{aligned} \theta^*(\lambda_A)(xy) &= \bigwedge_{z \in [xy]_\theta} \lambda_A(z) \\ &\leq \bigwedge_{z \in [x]_\theta [y]_\theta} \lambda_A(z) \\ &= \bigwedge_{ab \in [x]_\theta [y]_\theta} \lambda_A(ab) \\ &\leq \bigwedge_{a \in [x]_\theta, b \in [y]_\theta} [\lambda_A(a) \vee \lambda_A(b)] \\ &= \left(\bigwedge_{a \in [x]_\theta} \lambda_A(a) \right) \vee \left(\bigwedge_{b \in [y]_\theta} \lambda_A(b) \right) \\ &= \theta^*(\lambda_A)(x) \vee \theta^*(\lambda_A)(y) \end{aligned}$$

Therefore, $\theta^*(A)$ is an intuitionistic fuzzy subring of R .

Hence A is an upper rough intuitionistic fuzzy subring of R .

(b) Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of R .

Enough to prove that,

$$\theta^*(\mu_A)(xy) \geq \theta^*(\mu_A)(x) \vee \theta^*(\mu_A)(y) \text{ and}$$

$$\theta^*(\lambda_A)(xy) \leq \theta^*(\lambda_A)(x) \wedge \theta^*(\lambda_A)(y).$$

$$\text{Now, } \theta^*(\mu_A)(xy) = \bigvee_{z \in [xy]_\theta} \mu_A(z)$$

$$\begin{aligned} &\geq \bigvee_{z \in [x]_\theta [y]_\theta} \mu_A(z) \\ &= \bigvee_{ab \in [x]_\theta [y]_\theta} \mu_A(ab) \\ &\geq \bigvee_{a \in [x]_\theta, b \in [y]_\theta} (\mu_A(a) \vee \mu_A(b)) \\ &= \left(\bigvee_{a \in [x]_\theta} \mu_A(a) \right) \vee \left(\bigvee_{b \in [y]_\theta} \mu_A(b) \right) \\ &= \theta^*(\mu_A)(x) \vee \theta^*(\mu_A)(y) \end{aligned}$$

$$\text{and } \theta^*(\lambda_A)(xy) = \bigwedge_{z \in [xy]_\theta} \lambda_A(z)$$

$$\begin{aligned} &\leq \bigwedge_{z \in [x]_\theta [y]_\theta} \lambda_A(z) \\ &= \bigwedge_{ab \in [x]_\theta [y]_\theta} \lambda_A(ab) \\ &\leq \bigwedge_{a \in [x]_\theta, b \in [y]_\theta} (\lambda_A(a) \wedge \lambda_A(b)) \\ &= \left(\bigwedge_{a \in [x]_\theta} \lambda_A(a) \right) \wedge \left(\bigwedge_{b \in [y]_\theta} \lambda_A(b) \right) \\ &= \theta^*(\lambda_A)(x) \wedge \theta^*(\lambda_A)(y) \end{aligned}$$

This shows that $\theta^*(A)$ is an intuitionistic fuzzy ideal of R .

Therefore, A is an upper rough intuitionistic fuzzy ideal of R .

Theorem 3.12 Let θ be a complete congruence relation on R . Then

(a) If A is an intuitionistic fuzzy subring of R , then A is an lower rough intuitionistic fuzzy subring of R .

(b) If A is an intuitionistic fuzzy ideal of R , then A is an lower rough intuitionistic fuzzy ideal of R .

Proof. Since θ is complete congruence relation on R , we have

$$[a]_\theta - [b]_\theta = [a-b]_\theta \text{ and } [a]_\theta [b]_\theta = [ab]_\theta, \forall a, b \in R.$$

(a) Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy subring of R and $\theta_*(A) = (\theta_*(\mu_A), \theta_*(\lambda_A))$.

Now, for all $x, y \in R$, we have

$$\begin{aligned} \text{(i) } \theta_*(\mu_A)(x-y) &= \bigwedge_{z \in [x-y]_\theta} \mu_A(z) \\ &= \bigwedge_{z \in [x]_\theta - [y]_\theta} \mu_A(z) \\ &= \bigwedge_{a-b \in [x]_\theta - [y]_\theta} \mu_A(a-b) \\ &\geq \bigwedge_{a \in [x]_\theta, b \in [y]_\theta} (\mu_A(a) \wedge \mu_A(b)) \\ &= \left(\bigwedge_{a \in [x]_\theta} \mu_A(a) \right) \wedge \left(\bigwedge_{b \in [y]_\theta} \mu_A(b) \right) \\ &= \theta_*(\mu_A)(x) \wedge \theta_*(\mu_A)(y) \end{aligned}$$

$$\begin{aligned} \theta_*(\lambda_A)(x-y) &= \bigvee_{z \in [x-y]_\theta} \lambda_A(z) \\ &= \bigvee_{z \in [x]_\theta - [y]_\theta} \lambda_A(z) \\ &= \bigvee_{a-b \in [x]_\theta - [y]_\theta} \lambda_A(a-b) \\ &\leq \bigvee_{a \in [x]_\theta, b \in [y]_\theta} (\lambda_A(a) \vee \lambda_A(b)) \\ &= \left(\bigvee_{a \in [x]_\theta} \lambda_A(a) \right) \vee \left(\bigvee_{b \in [y]_\theta} \lambda_A(b) \right) \\ &= \theta_*(\lambda_A)(x) \vee \theta_*(\lambda_A)(y) \end{aligned}$$

$$\begin{aligned} \text{(ii) } \theta_*(\mu_A)(xy) &= \bigwedge_{z \in [xy]_\theta} \mu_A(z) \\ &= \bigwedge_{z \in [x]_\theta [y]_\theta} \mu_A(z) \\ &= \bigwedge_{ab \in [x]_\theta [y]_\theta} \mu_A(ab) \\ &\geq \bigwedge_{a \in [x]_\theta, b \in [y]_\theta} (\mu_A(a) \wedge \mu_A(b)) \\ &= \left(\bigwedge_{a \in [x]_\theta} \mu_A(a) \right) \wedge \left(\bigwedge_{b \in [y]_\theta} \mu_A(b) \right) \\ &= \theta_*(\mu_A)(x) \wedge \theta_*(\mu_A)(y) \end{aligned}$$

$$\theta_*(\lambda_A)(xy) = \bigvee_{z \in [xy]_\theta} \lambda_A(z)$$

$$\begin{aligned}
 &= \bigvee_{z \in [x]_{\theta} [y]_{\theta}} \lambda_A(z) \\
 &= \bigvee_{ab \in [x]_{\theta} [y]_{\theta}} \lambda_A(ab) \\
 &\leq \bigvee_{a \in [x]_{\theta}, b \in [y]_{\theta}} (\lambda_A(a) \vee \lambda_A(b)) \\
 &= \left(\bigvee_{a \in [x]_{\theta}} \lambda_A(a) \right) \vee \left(\bigvee_{b \in [y]_{\theta}} \lambda_A(b) \right) \\
 &= \theta_*(\lambda_A)(x) \vee \theta_*(\lambda_A)(y)
 \end{aligned}$$

Therefore, $\theta_*(A)$ is an intuitionistic fuzzy subring of R .

Hence A is an lower rough intuitionistic fuzzy subring of R .

(b) Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of R .

Enough to prove that,

$$\theta_*(\mu_A)(xy) \geq \theta_*(\mu_A)(x) \vee \theta_*(\mu_A)(y) \text{ and}$$

$$\theta_*(\lambda_A)(xy) \leq \theta_*(\lambda_A)(x) \wedge \theta_*(\lambda_A)(y).$$

$$\begin{aligned}
 \text{Now, } \theta_*(\mu_A)(xy) &= \bigwedge_{z \in [xy]_{\theta}} \mu_A(z) \\
 &= \bigwedge_{z \in [x]_{\theta} [y]_{\theta}} \mu_A(z) \\
 &= \bigwedge_{ab \in [x]_{\theta} [y]_{\theta}} \mu_A(ab) \\
 &\geq \bigwedge_{a \in [x]_{\theta}, b \in [y]_{\theta}} (\mu_A(a) \vee \mu_A(b)) \\
 &= \left(\bigwedge_{a \in [x]_{\theta}} \mu_A(a) \right) \vee \left(\bigwedge_{b \in [y]_{\theta}} \mu_A(b) \right) \\
 &= \theta_*(\mu_A)(x) \vee \theta_*(\mu_A)(y)
 \end{aligned}$$

$$\begin{aligned}
 \text{and } \theta_*(\lambda_A)(xy) &= \bigvee_{z \in [xy]_{\theta}} \lambda_A(z) \\
 &= \bigvee_{z \in [x]_{\theta} [y]_{\theta}} \lambda_A(z) \\
 &= \bigvee_{ab \in [x]_{\theta} [y]_{\theta}} \lambda_A(ab) \\
 &\leq \bigvee_{a \in [x]_{\theta}, b \in [y]_{\theta}} (\lambda_A(a) \wedge \lambda_A(b)) \\
 &= \left(\bigvee_{a \in [x]_{\theta}} \lambda_A(a) \right) \wedge \left(\bigvee_{b \in [y]_{\theta}} \lambda_A(b) \right) \\
 &= \theta_*(\lambda_A)(x) \wedge \theta_*(\lambda_A)(y)
 \end{aligned}$$

This shows that, $\theta_*(A)$ is an intuitionistic fuzzy ideal of R .

Therefore, A is an lower rough intuitionistic fuzzy ideal of R .

Corollary 3.13 Let θ be a complete congruence relation on R , A be a intuitionistic fuzzy subring (ideal) of R . Then

$\theta(A) = (\theta_*(A), \theta^*(A))$ is a rough intuitionistic fuzzy subring (ideal) of R .

4 ROUGH INTUITIONISTIC FUZZY PRIME IDEAL OF A RING

Definition 4.1 [3] For any intuitionistic fuzzy set

$A = \{ \langle x, \mu_A(x), \lambda_A(x) \rangle \mid x \in R \}$ of a set R , we define a

(α, β) -cut of A as the subset

$\{ x \in R \mid \mu_A(x) \geq \alpha, \lambda_A(x) \leq \beta \}$ of R and it is denoted by

$C_{\alpha, \beta}(A)$ and the strong (α, β) -cut of A is denoted by

$C_{\alpha, \beta}^S(A)$ and is defined as

$$C_{\alpha, \beta}^S(A) = \{ x \in R \mid \mu_A(x) > \alpha, \lambda_A(x) < \beta \}$$

Lemma 4.2 [3] If A is an intuitionistic fuzzy ideal of a ring R then

$C_{\alpha, \beta}(A)$ is an ideal of R if $\mu_A(0) \geq \alpha, \lambda_A(0) \leq \beta$ and

$\alpha + \beta \leq 1$.

Theorem 4.3 [3] If $C_{\alpha, \beta}(A)$ is an ideal of R for all $\alpha, \beta \in [0, 1]$

with $\alpha + \beta \leq 1$ then $A = \{ \langle x, \mu_A(x), \lambda_A(x) \rangle \mid x \in R \}$ is an intuitionistic fuzzy ideal of R .

Theorem 4.4 [3] If $f: R \rightarrow R'$ be an epimorphism and

$A = \{ \langle x, \mu_A(x), \lambda_A(x) \rangle \mid x \in R \}$ be an intuitionistic fuzzy

ideal of R then $f(C_{\alpha, \beta}(A)) \subseteq C_{\alpha, \beta}(f(A))$ and the equality

holds if μ_A has the sup property and $\alpha + \beta = 1$.

Theorem 4.5 [3] If $f: R \rightarrow R'$ be a homomorphism and

$A' = \{ \langle y, \mu_{A'}(y), \lambda_{A'}(y) \rangle \mid y \in R' \}$ be an intuitionistic fuzzy

ideal of R' then $f^{-1}(C_{\alpha, \beta}(A')) = C_{\alpha, \beta}(f^{-1}(A'))$.

Theorem 4.6 If $f: R \rightarrow R'$ be an epimorphism and

$A = \{ \langle x, \mu_A(x), \lambda_A(x) \rangle \mid x \in R \}$ be an intuitionistic fuzzy

ideal of R , μ_A has the sup property with $\alpha + \beta = 1$ then $f(A)$

is an intuitionistic fuzzy ideal of R' .

Proof. The proof follows from Theorem 4.4.

Theorem 4.7 If $f: R \rightarrow R'$ be a homomorphism and A' be an intuitionistic fuzzy ideal of R' then $f^{-1}(A')$ is a intuitionistic fuzzy

ideal of R .

Proof. The proof is straight forward.

Theorem 4.8 If A is an intuitionistic fuzzy prime ideal of a ring R then $C_{\alpha,\beta}(A)$ is an prime ideal of R if $\mu_A(1) < \alpha$ and $\lambda_A(0) < \beta$ with $\alpha + \beta \leq 1$.

Theorem 4.9 [10] (a) Let θ be a complete congruence relation on R and A a prime ideal of R such that $\theta^*(A) \neq R$, then A is an upper rough prime ideal of R .

(b) Let θ be a full congruence relation on R and A be a prime ideal of R . If $\theta_*(A) \neq \emptyset$, then A is a lower rough prime ideal of R .

Theorem 4.10 Let θ be a full congruence relation on R . If A is an intuitionistic fuzzy subset of R and $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$ then

- (i) $C_{\alpha,\beta}(\theta_*(A)) = \theta_*(C_{\alpha,\beta}(A))$
- (ii) $\theta^*(C_{\alpha,\beta}^S(A)) \subseteq C_{\alpha,\beta}^S(\theta^*(A))$. Equality holds if A has sup property and $\alpha + \beta = 1$.

Proof.

Let $A = (\mu_A, \lambda_A)$ be a IFS of R .

- (i) Suppose $x \in C_{\alpha,\beta}(\theta_*(A)) \Leftrightarrow x \in C_{\alpha,\beta}(\theta_*(\mu_A), \theta_*(\lambda_A))$

$$\Leftrightarrow \theta_*(\mu_A)(x) \geq \alpha \text{ and } \theta_*(\lambda_A)(x) \leq \beta$$

$$\Leftrightarrow \bigwedge_{a \in [x]_\theta} \mu_A(a) \geq \alpha \text{ and } \bigvee_{a \in [x]_\theta} \lambda_A(a) \leq \beta$$

$$\Leftrightarrow \mu_A(a) \geq \alpha \text{ and } \lambda_A(a) \leq \beta, \text{ for all } a \in [x]_\theta$$

$$\Leftrightarrow a \in C_{\alpha,\beta}(A), \text{ for all } a \in [x]_\theta$$

$$\Leftrightarrow [x]_\theta \subseteq C_{\alpha,\beta}(A)$$

$$\Leftrightarrow x \in \theta_*(C_{\alpha,\beta}(A))$$

$$\text{Thus, } C_{\alpha,\beta}(\theta_*(A)) = \theta_*(C_{\alpha,\beta}(A)).$$

- (ii) Let $x \in \theta^*(C_{\alpha,\beta}^S(A))$

$$\Rightarrow [x]_\theta \cap C_{\alpha,\beta}^S(A) \neq \emptyset$$

$$\Rightarrow \text{there exists } a \in R \text{ such that } a \in [x]_\theta \text{ and } a \in C_{\alpha,\beta}^S(A).$$

$$\Rightarrow \mu_A(a) > \alpha \text{ and } \lambda_A(a) < \beta, \text{ for some } a \in [x]_\theta$$

$$\Rightarrow \bigvee_{a \in [x]_\theta} \mu_A(a) > \alpha \text{ and } \bigwedge_{a \in [x]_\theta} \lambda_A(a) < \beta$$

$$\Rightarrow (\theta^*(\mu_A))(x) > \alpha \text{ and } (\theta^*(\lambda_A))(x) < \beta$$

$$\Rightarrow x \in C_{\alpha,\beta}^S(\theta^*(\mu_A), \theta^*(\lambda_A))$$

$$\Rightarrow x \in C_{\alpha,\beta}^S(\theta^*(A))$$

$$\text{Thus } \theta^*(C_{\alpha,\beta}^S(A)) \subseteq C_{\alpha,\beta}^S(\theta^*(A))$$

For the other part, let A has sup property and $\alpha + \beta = 1$.

$$\text{Now, } x \in C_{\alpha,\beta}^S(\theta^*(A))$$

$$\Rightarrow (\theta^*(\mu_A))(x) > \alpha \text{ and } (\theta^*(\lambda_A))(x) < \beta$$

$$\Rightarrow \bigvee_{a \in [x]_\theta} \mu_A(a) > \alpha \text{ and } \bigwedge_{a \in [x]_\theta} \lambda_A(a) < \beta$$

$$\Rightarrow \mu_A(z) = \bigvee_{a \in [x]_\theta} \mu_A(a) > \alpha, \text{ since } A \text{ has sup property}$$

$$\Rightarrow \mu_A(z) > \alpha, \text{ for some } z \in [x]_\theta \text{ and } \lambda_A(z) \leq 1 - \mu_A(z) < 1 - \alpha = \beta.$$

$$\text{Therefore, } z \in C_{\alpha,\beta}^S(A).$$

$$\Rightarrow [x]_\theta \cap C_{\alpha,\beta}^S(A) \neq \emptyset$$

$$\Rightarrow x \in \theta^*(C_{\alpha,\beta}^S(A))$$

$$\text{Therefore, } C_{\alpha,\beta}^S(\theta^*(A)) \subseteq \theta^*(C_{\alpha,\beta}^S(A)).$$

Hence the equality follows.

Theorem 4.11 Let A be an intuitionistic fuzzy prime ideal of R and $\alpha, \beta \in [0, 1], \alpha > \mu_A(1)$ and $\beta > \lambda_A(0)$ with $\alpha + \beta \leq 1$.

- (a) If θ is complete congruence relation on R and $\theta_*(A) \neq \emptyset$, then A is a lower rough intuitionistic fuzzy prime ideal of R .
- (b) If θ is complete congruence relation on R , μ_A has sup property and $\alpha + \beta = 1$, then A is an upper rough intuitionistic fuzzy prime ideal of R .

Proof.

- (a) Since A is a intuitionistic fuzzy prime ideal of R , by Theorem 4.8, we know that $C_{\alpha,\beta}(A) (\alpha > \mu_A(1), \beta > \lambda_A(0))$ is, if it is non-empty, a prime ideal of R .

Then by Theorem 4.9, we obtain that $\theta_*(C_{\alpha,\beta}(A))$, if it is non-empty, is a prime ideal of R .

From this and Theorem 4.10(i), we know that $C_{\alpha,\beta}(\theta_*(A))$ is a prime ideal of R .

Now, by Theorem 4.3, we obtain that $\theta_*(A)$ is a intuitionistic fuzzy prime ideal of R .

- (b) It can be seen in a similar way.

Theorem 4.12 Let θ be a complete congruence relation on R . Then A is a lower [upper] rough intuitionistic fuzzy prime ideal of R if and only if $C_{\alpha,\beta}^S(A), C_{\alpha,\beta}^S(A)$ are, if they are non-empty, lower [upper] rough prime ideals of R for every $\alpha, \beta \in [0, 1]$ with $\alpha + \beta = 1$ and μ_A has the sup property.

Proof. The proof follows from Theorem 4.10 and Theorem 4.11.

Theorem 4.13 [10] Let f be an epimorphism of a ring R_1 to a ring R_2 and let θ_2 be a full congruence relation on R_2 . Let A be a subset of R_1 . If $\theta_1 = \{(a, b) \in R_1 \times R_1 \mid (f(a), f(b)) \in \theta_2\}$, then

- (a) $\theta_1^*(A)$ is an ideal of R_1 if and only if $\theta_2^*(f(A))$ is an ideal of R_2 .
- (b) If θ_2 is complete, $\theta_1^*(A)$ is a prime ideal of R_1 if and only if $\theta_2^*(f(A))$ is a prime ideal of R_2 .

Theorem 4.14 [10] Let f be an isomorphism of a ring R_1 to a ring R_2 and let θ_2 be a complete congruence relation on R_2 . Let A be a subset of R_1 . If $\theta_1 = \{(a, b) \in R_1 \times R_1 \mid (f(a), f(b)) \in \theta_2\}$, then

- (a) $\theta_1^*(A)$ is an ideal of R_1 if and only if $\theta_2^*(f(A))$ is an ideal of R_2 .
- (b) $\theta_1^*(A)$ is a prime ideal of R_1 if and only if $\theta_2^*(f(A))$ is a prime ideal of R_2 .

Theorem 4.15 Let f be an epimorphism of a ring R_1 to a ring R_2 and let θ_2 be a complete congruence relation on R_2 . Let A be a intuitionistic fuzzy subset of R_1 . If $\theta_1 = \{(a, b) \in R_1 \times R_1 \mid (f(a), f(b)) \in \theta_2\}$, then

- (a) $\theta_1^*(A)$ is a intuitionistic fuzzy ideal of R_1 and μ_A has the sup property if and only if $\theta_2^*(f(A))$ is a intuitionistic fuzzy ideal of R_2 .
- (b) $\theta_1^*(A)$ is a intuitionistic fuzzy prime ideal of R_1 if and only if $\theta_2^*(f(A))$ is a intuitionistic fuzzy prime ideal of R_2 .

More over if f is one to one then we have,

- (c) $\theta_1^*(A)$ is a intuitionistic fuzzy ideal of R_1 if and only if

$\theta_2^*(f(A))$ is a intuitionistic fuzzy ideal of R_2 .

- (d) $\theta_1^*(A)$ is a intuitionistic fuzzy prime ideal of R_1 if and only if $\theta_2^*(f(A))$ is a intuitionistic fuzzy prime ideal of R_2 .

Proof.

- (a) By Theorem 3.11(b), we obtain that $\theta_1^*(A)$ is a intuitionistic fuzzy ideal of R_1 if and only if $C_{\alpha,\beta}^S(\theta_1^*(A))$ is, if it is nonempty, an ideal of R_1 for every $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$.

Using Theorem 4.10(i), we have $C_{\alpha,\beta}^S(\theta_1^*(A)) = \theta_1^*(C_{\alpha,\beta}^S(A))$. By Theorem 4.13(a), we obtain that $\theta_1^*(C_{\alpha,\beta}^S(A))$ is an ideal of R_1 if and only if $\theta_2^*(f(C_{\alpha,\beta}^S(A)))$ is an ideal of R_2 .

Therefore, we have

$$\theta_2^*(f(C_{\alpha,\beta}^S(A))) = \theta_2^*(C_{\alpha,\beta}^S(f(A))) = C_{\alpha,\beta}^S(\theta_2^*(f(A)))$$

By Theorem 3.11(b), we obtain $C_{\alpha,\beta}^S(\theta_2^*(f(A)))$ is an ideal of R_2 if and only if $\theta_2^*(f(A))$ is a intuitionistic fuzzy ideal of R_2 .

The proof of (b), (c), (d) is similar to the proof of (a).

5 CONCLUSION

The intuitionistic fuzzy sets of a set are a generalization of a fuzzy set in a set. In this paper, we have given the notion of rough intuitionistic fuzzy set in a ring and studied some of their properties. We also proved that any intuitionistic fuzzy subring (ideal) of a ring is an upper (lower) rough intuitionistic fuzzy subring (ideal) of the ring. Further we have proved that the homomorphic image of lower (upper) rough intuitionistic fuzzy ideal of a ring is also a lower (upper) intuitionistic fuzzy ideal. We hope that our results can also be extended to other algebraic systems.

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